From Functions to Circuits

…via truth tables…

Truth tables revisited

A truth table **uniquely defines** a Boolean function.

If we only have the truth table, how can we “extract” an expression for the function?

X	*Y*	*Z*	*F(X,Y,Z)*
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

The "Sum-of-Products" method

We want to find an expression that's TRUE exactly for the rows where *F(X,Y,Z)*=1.

X	*Y*	*Z*	*F(X,Y,Z)*
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

The “Sum-of-Products” method

Products

Sum

$$
\begin{aligned}
& \overline{X}Y\overline{Z} \\
+\ & \overline{X}YZ \\
+\ & X\overline{Y}\,\overline{Z} \\
+\ & XY\overline{Z} \\
+\ & XYZ
\end{aligned}
$$

X	*Y*	*Z*	*F(X,Y,Z)*
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

From Function to Circuit

So the function can be described by *F(X,Y,Z)*=

$$\overline{X}\,Y\,\overline{Z} + \overline{X}\,Y\,Z + X\,\overline{Y}\,\overline{Z} + X\,Y\,\overline{Z} + X\,Y\,Z$$

For any function in sum-of-products form, we can build a circuit in a very systematic way.

From Function to Circuit

$$
\begin{aligned}
& \overline{X}Y\overline{Z} \\
+ \; & \overline{X}YZ \\
+ \; & X\overline{Y}\,\overline{Z} \\
+ \; & XY\overline{Z} \\
+ \; & XYZ
\end{aligned}
$$

Equivalent expressions

So the function can be described by *F(X,Y,Z)*=

$$\overline{X}\,Y\,\overline{Z} + \overline{X}\,Y\,Z + X\,\overline{Y}\,\overline{Z} + X\,Y\,\overline{Z} + X\,Y\,Z$$

But what about the way we previously wrote it?

$$F(X,Y,Z) = X\overline{Z} + Y$$

The two are **equivalent**. We will see next how to simplify a function.

Summary

- Truth tables uniquely define Boolean functions.
- We can extract a “sum-of-products” form from a truth table by looking at the rows that result in 1
- Sums of products are easy to convert into circuits

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