

From Functions to Circuits

...via truth tables...

Truth tables revisited

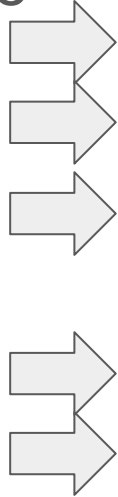
A truth table **uniquely defines** a Boolean function.

If we only have the truth table, how can we “extract” an expression for the function?

X	Y	Z	$F(X,Y,Z)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

The “Sum-of-Products” method

We want to find an expression that's TRUE exactly for the rows where $F(X, Y, Z)=1$.



X	Y	Z	$F(X,Y,Z)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

The “Sum-of-Products” method

Sum

Products

$$\begin{aligned} &+ \overline{X} \overline{Y} \overline{Z} \\ &+ \overline{X} Y Z \\ &+ X \overline{Y} \overline{Z} \\ &+ X Y \overline{Z} \\ &+ X Y Z \end{aligned}$$

X	Y	Z	$F(X,Y,Z)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

From Function to Circuit

So the function can be described by $F(X, Y, Z) =$

$$\overline{X} Y \overline{Z} + \overline{X} Y Z + X \overline{Y} \overline{Z} + X Y \overline{Z} + X Y Z$$

For any function in sum-of-products form, we can build a circuit in a very systematic way.

From Function to Circuit

$$\begin{aligned} & \overline{X}Y\overline{Z} \\ + & \overline{X}YZ \\ + & X\overline{Y}\overline{Z} \\ + & XY\overline{Z} \\ + & XYZ \end{aligned}$$

Equivalent expressions

So the function can be described by $F(X, Y, Z) =$

$$\overline{X} Y \overline{Z} + \overline{X} Y Z + X \overline{Y} \overline{Z} + X Y \overline{Z} + X Y Z$$

But what about the way we previously wrote it?

$$F(X, Y, Z) = X \overline{Z} + Y$$

The two are **equivalent**. We will see next how to simplify a function.

Summary

- Truth tables uniquely define Boolean functions.
- We can extract a “sum-of-products” form from a truth table by looking at the rows that result in 1
- Sums of products are easy to convert into circuits

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