Monash University
Faculty of Information Technology
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FIT1058
Sample Exam Solutions

QUESTION 1

(a) $A \setminus C$ is the simplest, though not the only correct, solution. $(A \cap B) \cap \overline{C}$ also works.

(b) $|(\overline{A \cup B \cup C})| = 3$.

By the Inclusion-Exclusion principle:
$|(A \cup B \cup C)|$
$= |A| + |B| + |C| - |(A \cap B)| - |(A \cap C)| - |(B \cap C)| + |(A \cap B \cap C)|$
$= 5 + 4 + 5 - 3 - 3 - 2 + 1 = 7.$

Then, finding the cardinality of the complement:
$|(\overline{A \cup B \cup C})| = |U| - |(A \cup B \cup C)| = 10 - 7 = 3.$

QUESTION 2

The statement $\overline{(A \cap B)} = \overline{A} \cap \overline{B}$ is **not always true**. (Remember De Morgan's Laws for sets!)

QUESTION 3

(a) 26^5

(b) An example function could be as follows:

$f : X \to Y$
$f(s) = n,$ where n is the position in the alphabet (from 1–26) of the first letter of s.

This is *surjective* because X includes all possible alphabetic strings of length 5, so there is at least one string starting with every letter of the alphabet. So every value in Y is mapped to at least once.

It is *not injective*, because X includes multiple strings that start with the same letter. So each value in Y is mapped to *more* than once.

(c) g is neither surjective nor injective. It is a **constant** function — every string in X is mapped to the number 5, since all strings in X have the same length.

QUESTION 4

If $g \circ f$ is injective, then f is injective.

QUESTION 5

R is reflexive, symmetric, and transitive.

QUESTION 6

$n^2+n=n(n+1)$

Since n and $n+1$ are consecutive integers, exactly one of them must be even.

Case 1: Suppose n is even, and $n+1$ is odd. Then we can write $n=2k$, for some integer k.

So $n(n+1)=2k(2k+1)=2(2k^2+k)$.

Since k is an integer, the expression $2k^2+k$ is also an integer, so 2 times this expression is even.

Case 2: Suppose n is odd, and $n+1$ is even. Then we can write $n=2k-1$, for some integer k.

So $n(n+1)=2(k-1)(2(k-1)+1)=(2k-2)(2k-1)=4k^2-6k+2=2(k^2-3k+1)$.

Since k is an integer, the expression k^2-3k+1 is also an integer, so 2 times this expression is even.

So in either case, $n(n+1)$ is even.

QUESTION 7

$P \Rightarrow (Q \Rightarrow R)$

$=$	$\neg P \vee (\neg Q \vee R)$	($A \Rightarrow B$ is equivalent to $\neg A \vee B$, applied twice)
$=$	$\neg P \vee \neg Q \vee R$	(removing brackets)
$=$	$\neg(P \wedge Q) \vee R$	(De Morgan's Law on first 2 terms)
$=$	$(P \wedge Q) \Rightarrow R$	($\neg A \vee B$ is equivalent to $A \Rightarrow B$)

QUESTION 8

(a) $P(\text{Java}) \wedge C(\text{Java})$

(b) $\forall X : P(X) \Rightarrow (C(X) \vee I(X))$

(c) $\exists X (P(X) \wedge \forall Y ((P(Y) \wedge X \neq Y) \Rightarrow N(X,Y)))$

QUESTION 9

(a) $a_2 = 2 \times 3 + 1 = 7$

$a_3 = 2 \times 7 + 1 = 15$

$a_4 = 2 \times 15 + 1 = 31$

(b) Closed form: $a_n = 2^{n+1} - 1$

(c) **Base case:** $a_1 = 3$ (from base case of recurrence relation)

Inductive step:

For $k \geq 1$, assume $a_k = 2^{k+1} - 1$. (Inductive hypothesis)

Then: $a_{k+1} = 2a_k + 1$ (from recurrence relation)

$= 2(2^{k+1} - 1) + 1$ (replacing a_k term, by inductive hypothesis)

$= 2^{k+2} - 2 + 1$ (multiplying out)

$= 2^{(k+1)+1} - 1$

So $a_{k+1} = 2^{(k+1)+1} - 1$, as required.

Therefore, by the principle of mathematical induction, $a_n = 2^{n+1} - 1$ for all $n \geq 1$.

QUESTION 10

(a) $f_n = 2 + 3(n-1) = 3n - 1$

(b) $T(n) = na + (n(n-1)/2)d = 2n + 3n(n-1)/2$

(c) $O(n^2)$

QUESTION 11

(a) $120 = 23 \times 3 \times 5$

So the distinct primes are: $2, 3, 5$.

$\phi(120) = 120(1 - 1/2)(1 - 1/3)(1 - 1/5)$

$= 120 \cdot (1/2) \cdot (2/3) \cdot (4/5)$

$= 60 \cdot (2/3) \cdot (4/5)$

$= 40 \cdot (4/5)$

$= 32$

(b) Input: $m = 90, n = 42$ (since m must be $\geq n$)

$(a, x, y) = (90, 1, 0); (b, z, w) = (42, 0, 1)$; continue

$(a, x, y) = (42, 0, 1); (b, z, w) = (90, 1, 0) - 2(42, 0, 1) = (6, 1, -2)$; continue

$(a, x, y) = (6, 1 - 2); (b, z, w) = (42, 0, 1) - 7(6, 1, -2) = (0, -7, 15)$; stop

$x = 1, y = -2$

Swap: $x = -2, y = 1$

$-2 \cdot 42 + 1 \cdot 90 = 6$

QUESTION 12

For each valid pair m, c:

Choose m maths textbooks from 5: $\binom{5}{m}$

Choose c CS textbooks from 6: $\binom{6}{c}$

Arrange all selected books on the shelf: 8!

So for each valid (m, c), the number of arrangements is $\binom{5}{m} \cdot \binom{6}{c} \cdot 8!$

Valid pairings are:

- $m = 2, c = 6$
- $m = 3, c = 5$
- $m = 4, c = 4$

- $m = 5, c = 3$

So the final expression is:

$$8! \cdot (\binom{5}{2}\binom{6}{6} + \binom{5}{3}\binom{6}{5} + \binom{5}{4}\binom{6}{4} + \binom{5}{5}\binom{6}{3})$$

QUESTION 13

Let:

- S be the event that a key is secure
- $\overline{S}$ be the event that a key is *not* secure
- T be the event that the test classifies a key as secure

We're given:

- $\Pr(S) = 0.7 \Rightarrow Pr(\overline{S}) = 0.3$
- $\Pr(T|S) = 0.90$ (correctly identifies secure keys)
- $\Pr(T|\overline{S}) = 0.20$ (false positive: classifies insecure keys as secure)

(a) Using the law of total probability:
$\Pr(T) = \Pr(T|S) \cdot \Pr(S) + \Pr(T|\overline{S}) \cdot \Pr(\overline{S})$
Substitute the values:
$\Pr(T) = (0.9)(0.7) + (0.20)(0.3) = 0.63 + 0.06 = 0.69$

(b) Using Bayes' Theorem:
$\Pr(S|T) = (\Pr(T|S) \cdot \Pr(S))/\Pr(T)$
Substitute the values:
$\Pr(T|S) = 0.90$
$\Pr(S) = 0.7$
$\Pr(T) = 0.69$ (from part a)
$\Pr(S|T) = (0.9 \cdot 0.7)/0.69 = 0.63/0.69 = 63/69 = 21/23$

QUESTION 14

Uniform distribution.

QUESTION 15

The expected value is 2.9.

QUESTION 16

(a) $X \sim \text{Geometric}(p = 0.2)$

The geometric distribution models the number of trials until the first success, with success probability p.

(b) $\Pr(X = 4) = (1-p)^3 \cdot p = 0.8^3 \cdot 0.2$

(c) $E(X) = 1/p = 1/0.2 = 5$

QUESTION 17

By the Handshaking Lemma, the sum of the degrees is 2 times the number of edges $= 2 \times 20 = 40$. So the average degree is $40/8 = 5$

QUESTION 18

Suppose for some connected graph G with 8 vertices, where each vertex has degree at least 5, that G is planar.

Let m be the number of edges in G.

Since each of the 8 vertices has degree at least 5, the sum of the degrees of all the vertices is $\geq (8 \times 5) = 40$. So by the Handshaking Lemma, $2m \geq 40 \Rightarrow m \geq 20$.

By Euler's inequality for planar graphs, m is at most $3v - 6 = 3 \times 8 - 6 = 18$.

So we have: $m \geq 20$, but we also have $m \leq 18$. This is a contradiction.

Conclusion: Our assumption that such a simple connected planar graph exists must be false.